4052

B. Com. (First Year) EXAMINATION

BUSINESS MATHS

Second Paper

Time : Three Hours *Maximum Marks : 80*

Note : Q. No. **1** is compulsory. Attempt *Five* questions including Q. No. **1**. All questions carry equal marks.

1. (a) Evaluate :

$$\int_1^2 \frac{x}{\sqrt{x-1}}\,dx$$

(b) Without expanding prove :

$$\begin{vmatrix} 3 & 1 & 6 \\ 5 & 2 & 10 \\ 7 & 4 & 14 \end{vmatrix} = 0$$

(c) Construct a 2×2 matrix, whose elements are given by $a_{ij} = \left(\frac{i-j}{2}\right)^2$.

(d) Divide 14 into two parts such that their product is maximum.

(e) Write advantages of LPP.

(f) Find the amount of an ordinary annuity of Rs. 600 payable at the end of each quarter for 2 years at 8% p.a. compounded quarterly.

(g) Solve $x+2y=5$ and $2x-y=5$ by determinants.

(h) Draw the graph of :

$$x+2y=5,\ 2x-y=5,\ x\geq 0,\ y\geq 0$$

2. (a) If $u=f(r)$ and $x^2+y^2=r^2$, prove that :

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$$

(b) Find the minimum value of $x^2+y^2+z^2$ subject to $xyz=a^3$.

3. (a) Evaluate :

$$\int \frac{x+1}{(x+2)^2}e^x\,dx$$

(b) Find the area of the region included between the parabolas $y^2=4ax$ and $x^2=4ay$, $a>0$.

4. (a) Prove that :

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

(b) Solve by matrix method :

$$x+y+z-7=0$$

$$x+2y+3z-16=0$$

$$x+3y+4z-22=0$$

5. (a) The population of a city increases every year by 2.3% of the population at the beginning of that year, in what time will the population treble itself ?

(b) Find the amount of an ordinary of Rs. 2,000 payable at the end of each year for 10 years, if money is worth 4% rate of interest per year.

6. Two labourers A and B earn Rs. 150 and Rs. 200 per day, respectively. A can manufacture 6 chairs and 4 tables, while B can manufacture 10 chairs and 4 tables per day. Form a LPP to minimize the labour cost to produce at least 60 chairs and 32 tables. Also find the minimum labour cost.

7. Solve the LPP :

Max. $Z = 2x_1 + 3x_2$,

subject to the constraints :

$$x_1 + 3x_2 \leq 12$$
$$2x_1 + x_2 \geq 6$$
$$x_1 + 5x_2 = 10$$
$$x_1, x_2 \geq 0$$

by finding the dual.

8. (a) If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}.

(b) Show that :

$$\int_3^{15} \frac{dx}{(x-3)\sqrt{x+1}} = \frac{1}{2}\log\frac{5}{3}$$

9. (a) If $u = f(x-y, y-z, z-x)$, prove that :

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

(b) A sum of money invested at C-I amounts to Rs. 10,816 at the end of second year and Rs. 11,248.64 at the end of third year. Find the rate of interest and the sum invested.