

Roll No....

(07/22-II)

5180

B.A./B.A.(Hons.)/B.Sc. EXAMINATION

(Second Semester)

MATHEMATICS

BM-122

Ordinary Differential Equations

Time : Three Hours Maximum Marks : $\begin{cases} \text{B.Sc. : 40} \\ \text{B.A. : 26} \end{cases}$

Note : Attempt Five questions in all, selecting one question from each Unit. Q. No. 1 is compulsory.

(Compulsory Question)

1. (a) Find the integrating factor of the equation

$$2\cos(y^2)dx - xys\sin(y^2)dy = 0. \quad 2(1)$$

(b) To show that $\frac{1}{D-\alpha}X = e^{\alpha x} \int (e^{-\alpha x} X) dx$, no arbitrary constant being added. 2(1)

(c) In what condition $y = x^m$ is a solution of the equation $\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = 0$? 2(1)

(d) Show that the differential equation :

$$(yz + 2x)dx + (zx - 2z)dy + (xy - 2y)dz = 0$$

is exact. 2(1)

Unit I

2. (a) Find the necessary and sufficient conditions that the equation $Mdx + Ndy = 0$ may be exact. 4(3)

- (b) Solve :

$$(xy^2 + 2x^2y^3)dx + (x^2y - x^3y^2)dy = 0.$$

4(2½)

3. (a) Solve the differential equation :

$$x^2 \left(\frac{dy}{dx} \right)^2 - 2xy \frac{dy}{dx} + 2y^2 - x^2 = 0. \quad 4(3)$$

(b) Obtain the complete primitive and singular solution of the equation :

$$x \left(\frac{dy}{dx} \right)^2 - 2y \frac{dy}{dx} + 4x = 0. \quad 4(2\frac{1}{2})$$

Unit II

4. (a) Find the orthogonal trajectories of the cardioid $r = a(1 - \cos\theta)$, where a is a parameter. $4(3)$

(b) Solve the differential equation :

$$\frac{d^3y}{dx^3} + y = 3 + e^{-x} + 5e^{2x}. \quad 4(2\frac{1}{2})$$

5. (a) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = xe^x \sin x. \quad 4(3)$$

(b) Solve the differential equation :

$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 2 \log x. \quad 4(2\frac{1}{2})$$

Unit III

6. (a) Solve the differential equation :

$$x^2 \frac{d^2y}{dx^2} - (x^2 + 2x) \frac{dy}{dx} + (x + 2)y = x^3 e^x. \quad 4(3)$$

(b) Solve the differential equation :

$$\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}$$

by variation of parameters method. $4(2\frac{1}{2})$

7. (a) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + (4x^2 - 3)y = e^{x^2}$$

by removing the first derivative. $4(3)$

(b) Solve :

$$\cos x \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} + 2y \cos^3 x = 2 \cos^5 x.$$

4(2½)

Unit IV

8. (a) Solve the simultaneous differential equations :

$$\begin{aligned}\frac{dx}{dt} + 5x + y &= e^t \\ \frac{dy}{dt} - x + 3y &= e^{2t}.\end{aligned}\quad 4(3)$$

(b) Solve the differential equations :

$$\frac{dx}{x+y} = \frac{dy}{-(x+y)} = \frac{dz}{z}. \quad 4(2\frac{1}{2})$$

9. (a) Solve the differential equation :

$$(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0.$$

4(3)

(b) Solve the differential equation :

$$(yz + z^2)dx - xzdy + xydz = 0. \quad 4(2\frac{1}{2})$$