Roll No. ....

(07/21-II)

5181

B.A./B.A. (Hons.)/B.Sc. EXAMINATION

(For Batch 2011 & Onwards)

(Second Semester)

MATHEMATICS

BM-123

Vector Calculus

Time : Three Hours *Maximum Marks :* B.Sc. : 40, B.A. : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1\. (a) Find the value of λ, so that the vectors $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \lambda\hat{i} + 2\hat{j} - 7\hat{k}$ and $\vec{c} = 5\hat{i} + 6\hat{j} - 5\hat{k}$ are co-planar. 1½(1)

(b) Prove that $\nabla\phi . d\vec{r} = d\phi$. 1½(1)

(c) Write the formulas of $\nabla . \vec{f}$ and $\nabla \times \vec{f}$ in terms of orthogonal curvilinear co-ordinates u, v and w. 1½(1)

(d) Evaluate :

$$\int_2^3 \left(\vec{r} . \frac{d\vec{r}}{dt} \right) dt,$$

where $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$ at $t = 2$ and $\vec{r} = 4\hat{i} - 2j + 3\hat{k}$ at $t = 3$. 1½(1)

(e) Find the unit normal vector to the surface $x^4 - 3xyz + z^2 + 1 = 0$ at the point (1, 2, 3). 2(1)

Section I

2\. (a) Show that the vectors $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ are non-coplanar if the vectors $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar. 4(3)

(b) If $\hat{r}$ is a unit normal vector in the direction of $\vec{r}$, then prove that : 4(2½)

$$\hat{r} \times \frac{d\hat{r}}{dt} = \frac{1}{r^2}\left(\vec{r} \times \frac{d\vec{r}}{dt}\right)$$

3. (a) If $\vec{a}', \vec{b}', \vec{c}'$ denote the reciprocal triad of vectors, prove that : 4(3)

$$\vec{a} \times \vec{a}' + \vec{b} \times \vec{b}' + \vec{c} \times \vec{c}' = \vec{0}$$

(b) Given $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$. Prove that : 4(2½)

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]\left[\vec{a}'\,\vec{b}'\,\vec{c}'\right] = 1.$$

Section II

4. (a) Find the angle of intersection at (4, –3, 2) of spheres $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z - 4 = 0$. 4(3)

(b) Evaluate $\nabla.(\vec{r} \times \vec{a})$, where $\vec{a}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{i} + z\hat{k}$. 4(2½)

5. (a) Prove that $\nabla \times (\phi\vec{f}) = \nabla\phi \times \vec{f} + \phi(\nabla \times \vec{f})$, where $\vec{f}$ and $\vec{g}$ are two vectors point function and ϕ is a scalar point function. 4(3)

(b) If $\vec{f} = \vec{\omega} \times \vec{r}$, then prove that $\vec{\omega} = \frac{1}{2}(\nabla \times \vec{f})$, where $\vec{\omega}$ is a constant vector. 4(2½)

Section III

6. (a) If u, v and w are orthogonal curvilinear co-ordinates, then prove that $\frac{\partial\vec{r}}{\partial u}, \frac{\partial\vec{r}}{\partial v}, \frac{\partial\vec{r}}{\partial w}$, and $\nabla u, \nabla v, \nabla w$ are reciprocal system of vectors. 4(3)

(b) Express $x\hat{i} + 2y\hat{j} + yz\hat{k}$ in spherical co-ordinates. 4(2½)

7. (a) Express the velocity $\vec{v}$ and acceleration $\vec{a}$ of a particle in cylindrical co-ordinates. 4(3)

(b) If $u = 3x + 2$, $v = y + 3$ and $w = z + 2$, show that u, v and w are orthogonal. Also find $(ds)^2$, h_1, h_2 and h_3. 4(2½)

Section IV

8. (a) Evaluate :

$$\int_C \vec{f}.d\vec{r},$$

where $\vec{f} = \left(x^2 + y^2\right)\hat{i} - 2xy\hat{j}$, the curve C is the rectangle in xy-plane bounded by $y = 0$, $x = a$, $y = b$ and $x = 0$. 4(3)

(b) If $\vec{f} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, evaluate $\iint_S \vec{f}.\hat{n}dS$, where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. 4(2½)

9. (a) Show that :

$$\iint_S \vec{f}.\hat{n}dS, = \iiint_V A^2 dV,$$

where $\vec{f} = \phi\vec{A}$, $\vec{A} = \nabla\phi$ and $\nabla^2\phi = 0$. 4(3)

(b) If ϕ and ψ are two scalar point functions having continuous second order derivatives in a region V bounded by closed surface S, then : 4(2½)

$$\iiint_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right)dV = \iint_S \left(\phi\nabla\psi - \psi\nabla\phi\right).\hat{n}dS$$

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