

Roll No. ....

(01/22-II)

5200

B.A./B.A. (Hons.)/B.Sc. EXAMINATION

(Third Semester)

MATHEMATICS

BM-232

Partial Differential Equations

Time : Three Hours Maximum Marks :  $\begin{cases} \text{B.Sc. : 40} \\ \text{B.A. : 26} \end{cases}$

Note : The candidates are required to attempt five questions in all, selecting the compulsory Q. No. 1 and one question from each Unit. Marks in brackets are for B. A. students.

Compulsory Question

1. (a) Define singular solution and general solution of PDE.

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(b) Solve  $(D^4 + D'^4)z = 0$ .

1½(1)

(c) Classify  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ . 1½(1)

(d) Define Laplace equation and wave equation. 1½(1)

(e) Solve  $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$ . 1½(1)

Unit I

2. (a) Find the partial differential equation of all spheres whose centre lie on z-axis. 4(2½)

(b) Solve  $(x^2 + 2y^2)p - xyq = xz$ . 4(2½)

3. (a) Find complete integral of

$x(1+y)p = y(1+x)q$ . 4(2½)

(b) Find complete integral of  $px + qy = pq$  using Charpit's method. 4(2½)

## Unit II

4. (a) Solve :

$$\left( D^3 - 4D^2 D' + 5DD'^2 - 2D'^3 \right) z = e^{2x+y}. \quad 4(2\frac{1}{2})$$

(b) Solve :

$$\left( D^2 - DD' + D' - 1 \right) z = \cos(x+2y). \quad 4(2\frac{1}{2})$$

5. (a) Solve :

$$\left( x^2 D^2 + 2xy DD' + y^2 D'^2 \right) z = x^2 y^2. \quad 4(2\frac{1}{2})$$

(b) Solve :

$$\left( 3D^2 - 2D'^2 + D'^{-1} \right) z = 4e^{x+y} \cos(x+y). \quad 4(2\frac{1}{2})$$

## Unit III

6. Reduce  $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$  to canonical form. 8(5)

7. Solve :  $x^{-2}r + y^{-2}t = x^{-3}p - y^{-3}q$ . 8(5)

## Unit IV

8. Solve the Cauchy problem for the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{1}{C^2} \cdot \frac{\partial^2 z}{\partial t^2} = 0, \quad C > 0 \quad \text{subject to the}$$

conditions  $z(x,0) = f(x)$  and  $\left[ \frac{\partial z}{\partial t} \right]_{t=0} = g(x)$

9. (a) Solve the equation  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t}$  with the conditions :

$$u(0,t) = u(l,t) = 0, \quad u(x,0) = x(l-x). \quad 4(2\frac{1}{2})$$

(b) Describe the method of separation of variables to find the solution of Laplace equation. 4(2 $\frac{1}{2}$ )