Roll No. ....

(06/21-II)

5219

B. A./B. A. (Hons.)/B. Sc.

EXAMINATION

(For Batch 2011 & Onwards)

(Fourth Semester)

MATHEMATICS

BM-241

Sequence and Series

Time : Three Hours *Maximum Marks :* B.Sc. : 40, B.A. : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory.

(Compulsory Question)

1. (a) Least upper bound (*l. u. b.*) of a set, it exists, is unique. 2(1)

(b) By definition show that $< \frac{1}{n^2} >$ converges to zero. 1½(1)

(c) Show that $\sum_{n=1}^{\infty} \sin \frac{1}{n^2}$ converges. 1½(1)

(d) Define absolute and conditional convergence. 1½(1)

(e) If the product $\prod_{n=1}^{\infty}(1+a_n)$ is convergent, then $\lim_{n\to\infty} a_n = 0$. 1½(1)

Unit I

2. (a) For any real number x, then exists unique integer n s.t. $n \leq x < n + 1$. 4(3)

(b) Every non-empty bounded below subset of real numbers has the greatest lower bound. 4(2½)

3. (a) Infinite union of closed sets may or may not be closed, which is clear by two examples (i.e., give *two* examples). 4(3)

(b) A set A is compact iff every open cover of A has a finite subcover. 4(2½)

Unit II

4. (a) If $\langle a_n \rangle$ is a sequence, then prove that $a_n \to 0$ iff $|a_n| \to 0$. 4(3)

(b) Using Squeeze principle, show that :

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots+\frac{1}{\sqrt{n^2+n}}\right]=1.$$ 4(2½)

5. (a) State and prove Cauchy's first theorem on limits. 4(3)

(b) Prove that the sequence $\langle a_n \rangle$ defined by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2a_n}$ converges to 2. 4(2½)

Unit III

6. (a) Test the convergence or divergence of

$$1+\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+\ldots, \text{ when } x > 0. \quad 4(3)$$

(b) The series :

$$\sum_{n=1}^{\infty}\frac{1}{np}=1+\frac{1}{2p}+\frac{1}{3p}+\ldots+\frac{1}{np}+\ldots$$

is (i) convergent if $p > 1$ and (ii) divergent if $p \leq 1$. 4(2½)

7. (a) Discuss the convergence of the series :

$$\frac{\alpha}{\beta}+\frac{1+\alpha}{1+\beta}+\frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)}+\ldots\ldots \quad 4(3)$$

(b) Using Cauchy's condensation test, discuss the convergence of the series :

$$\sum_{n=3}^{\infty}\frac{1}{n\log n(\log\log n)}. \quad 4(2½)$$

Unit IV

8. (a) Test the convergence and absolute convergence of the series :

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sin n\alpha}{n^2}, \ \alpha \text{ is real.}$$ 4(3)

(b) Test the convergence of the series :

$$\sum_{n=1}^{\infty} \frac{\cos nx}{np}, \text{ where } p > 0.$$ 4(2½)

9. (a) Show that $\prod_{n=0}^{\infty}\left(1+x^{2n}\right)$ converges to $\frac{1}{1-x}$ if $|x|<1$.

(b) Every absolutely convergent infinite product is convergent. 4(2½)