

Roll No.

(05/16-I)

5220

B. A./B. Sc. EXAMINATION

(Fourth Semester)

MATHEMATICS

Special Foundations & Integral Transforms

BM-242

Time : Three Hours Maximum Marks : { B.Sc. : 40
B.A. : 26

Note : Attempt Five questions in all, selecting one question from each Unit. Q. No. 1 is compulsory. Marks in bracket are shown for B.A

1. (a) Find the ordinary points and regular singular points of the differential equation. (2)(2)

$$(x-1)(x+2) \frac{d^2y}{dx^2} + x^2 \frac{dy}{dx} + xy = 0$$

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P.T.O.

(b) Show that : 1½(1)

$$[J_{1/2}^2(x) + J_{-1/2}^2(x)] = \frac{2}{\pi x}$$

(c) Using Rodrigue formula, find P₃(x). 1½(1)

(d) Find : 1½(1)

$$L^{-1} \left(\frac{1}{s} \sin \frac{1}{s} \right)$$

(e) If Fourier transform of f(x) is $\bar{f}(x)$, then prove Fourier transform of f(ax) is $\frac{1}{a} \bar{f} \left(\frac{s}{a} \right)$. 1½(1)

Unit I

2. (a) Find the series solution of the following differential equation about 0: 4(2½)

$$x(1-x) \frac{d^2y}{dx^2} + (1-5x) \frac{dy}{dx} - 4y = 0$$

(b) Show that : 4(2½)

$$\frac{d}{dx} (x^{-n} J_n(x)) = -x^{-n} J_{n+1}(x)$$

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3. (a) Use Jacobi's series to show that : 4(2½)

$$[J_0(x)]^2 + 2[J_1(x)]^2 + 2[J_2(x)]^2 + \dots = 1$$

(b) Find the solution of the following equation in terms of Bessel's function :

$$\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} + 4 \left(x^2 - \frac{n^2}{x^2} \right) y = 0 \quad 4(2\frac{1}{2})$$

Unit II

4. (a) State and prove orthogonality of Legendre Polynomial. 4(2½)

(b) Express $x^3 + 2x^2 - x - 3$ in terms of Legendre polynomial. 4(2½)

5. (a) Define Hermite's equation and show that :

$$H_n(x) = 2^n \left[\exp \left(-\frac{1}{4} \frac{d^2}{dx^2} \right) \right] x^n \quad 4(2\frac{1}{2})$$

(b) Expand e^{2x} in a series of Hermite's Polynomial. 4(2½)

Unit III

6. (a) Evaluate : 4(2½)

$$\int_b^a \frac{\sin mt}{t} dt$$

(b) Find : 4(4½)

$$L \left(\log \left(\frac{s^2 + 1}{(s-1)^2} \right) \right)$$

7. (a) Solve : 4(2½)

$$t \frac{d^2 y}{dt^2} + (1-2t) \frac{dy}{dt} - 2y = 0, \quad y(0) = 1$$

$y'(0) = 2$ using transform method.

(b) Solve : 4(2½)

$$f'(t) = t + \int_0^t f(t-u) \cos u \, du,$$

given $f(0) = 4$.

Unit IV

8. (a) Find the Fourier sine transform of $\frac{e^{-m}}{x}$. 4(2½)

(b) Using Parseval's identity, show that :

$$\int_0^{\infty} \frac{dx}{(x^2 + 25)(x^2 + 81)} = \frac{11}{1260} \quad 4(2\frac{1}{2})$$

9. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ given that $u(0, t) = 0$,

$u(\pi, t) = 0$ and $u(x, 0) = 2x$ when
 $0 < x < \pi, t > 0.$ 4(2½)

(b) Find the Fourier transforms of : 4(2½)

$$f(t) = \begin{cases} t^2, & |t| < t_0 \\ 0, & |t| > t_0 \end{cases}$$

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