Roll No. ....

(05/23-II)

5220

B.A./B.A.(Hons.)/B.Sc. EXAMINATION

(Fourth Semester)

MATHEMATICS

BM-242

Special Functions and Integral Transforms

Time : Three Hours *Maximum Marks :* B.Sc. : 40, B.A. : 26

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. Marks in brackets are for B.A. students.

(Compulsory Question)

1\. (a) Define Bessel function and its one property. 2(2)

P.T.O.

Define Beta and Gamma function. 1½(1)

(c) Define generating function Legendre function. 1½(1)

(d) Define shifting property of Fourier transform. 1½(1)

(e) State convolution property of Fourier transform. 1½(1)

Unit I

2\. Find the power series solution of the differential equation :

$$\frac{d^2y}{\partial x^2}+x\frac{dy}{dx}+x^2y=0 \text{ about `0'}. \quad 8(5)$$

3\. Solve the equation $x\frac{d^2y}{\partial x^2}+2\frac{dy}{dx}+\frac{1}{2}xy=0$ in terms of Bessel's function. 8(5)

Unit II

4\. (a) Prove that :

$$(2n+1)P_n(x)+P'_{n-1}(x)=P'_{n+1}(x). \quad 4(2½)$$

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(b) Prove that :

$$\int_{-1}^{1} P_m(x) P_n(x) dx = 0, \text{ when } m \neq n.$$ 4(2½)

5. (a) Describe generating function for Hermite ploynomial. 4(2½)

(b) Express $x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Hermite's polynomials. 4(2½)

Unit III

6. (a) Evaluate :

$$L\left[\int_0^t \frac{\sin t}{t} dt\right].$$ 4(2½)

(b) Evaluate :

$$\int_0^\infty te^{-2t} \sin t \, dt.$$ 4(2½)

7. (a) Find inverse Laplace transform of :

$$\frac{s+3}{(s+3)^2+4}.$$ 4(2½)

(b) Solve the differential equation by Laplace transform $\frac{d^2y}{dt^2} + y = t$, where $y(0) = 1$, $y'(0) = -2$. 4(2½)

Unit IV

8. (a) Find the Fourier transform of the function $f(x)$, where $f(x) = \begin{cases} x^2; & |x| < x_0 \\ 0; & |x| > x_0 \end{cases}$.

4(2½)

(b) State and prove convolution theroem for Fourier transform. 4(2½)

9. (a) State and prove Parseval's identity for Fourier transform. 4(2½)

(b) Solve :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},$$

subject to :

(i) $u(0, t) = 0$

(ii) $u(x, 0) = \begin{cases} 1; & 0 < x < 1 \\ 0; & x \geq 1 \end{cases}$

(iii) $u(x, t)$ is bounded. 4(2½)