Roll No. ....

(01/21-I)

5240

B.A./B.Sc. EXAMINATION

(Fifth Semester)

MATHEMATICS

BM-352

Group and Rings

Time : Three Hours *Max. Marks :* $\begin{cases} \text{B.Sc.} : 40 \\ \text{B.A.} : 26 \end{cases}$

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

(a) If a finite group of order 'n' contains an element of order 'n', then prove that the group must be cyclic. 2(2)

P.T.O.

(b) Show that every quotient group of an abelian group is abelian. 1½(1)

(c) Prove that subring of a commutative ring is commutative. 1½(1)

(d) Define kernel of a ring homomorphism. 1½(1)

(e) Express $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 3 & 4 & 2 & 1 \end{pmatrix}$ as the product of disjoint cycles. 1½(1)

Section I

2. (a) If a multiplicative group G has four elements, show that it must be abelian. 4(2½)

(b) If G = {0, 1, 2, 3, 4, 5} and binary operation is addition modulo 6, then prove that G is a cyclic group and find its generators. 4(2½)

3. (a) Use Lagrange's theorem to show that any group of prime order can have no proper subgroups. 4(2½)

(b) A subgroup H of G is normal subgroup if and only if the product of two right cosets of H in G is again a right coset of H in G. 4(2½)

Section II

4. (a) Let $f: G \to G$ be a homomorphism. Let f commutes with every inner automorphism of G. Show that $H = \{x \in G \mid f^2(x) = f(x)\}$ is a normal subgroup of G. 4(2½)

(b) Let Z(G) be the centre of a group G. If G/Z is cyclic, then prove that G is abelian. 4(2½)

5. (a) Find the centre of permutation group S_3. 4(2½)

(b) If N is a normal subgroup of a group G and $N \cap G' = \{e\}$, then show that $N \subseteq Z(G)$, where G′ is derived subgroup and Z(G) is centre of G. 4(2½)

Section III

6. (a) Prove that {0, 3, 6, 9} is a subring of the ring $(Z_{12}, +_{12}, X_{12})$. 4(2½)

(b) Define a division ring (skew field) and prove that it has no zero divisors. 4(2½)

7. (a) Prove that the ring of integers is a principal ideal ring. 4(2½)

(b) Let $f: R \to R'$ be a homomorphism. Then prove that f is one to one from R into R′, if and only if $\ker f = \{0\}$. 4(2½)

Section IV

8. (a) Prove that every ideal of an Euclidean ring is a principal ideal. 4(2½)

(b) Show that if an ideal S of a commutative ring R with unity element contains a unit of R, then S = R. 4(2½)

9\. (a) Prove that if R is an integral domain then R[x] is also an integral domain. 4(2½)

(b) Show that the polynomial $8x^3 - 6x - 1$ is irreducible over Q, the set of rational numbers. 4(2½)