(96/21-II)

5261

B.A./B.Sc. EXAMINATION

(For Batch 2011 & Onwards)

(Sixth Semester)

MATHEMATICS

BM-603

Dynamics

Time : Three Hours Maximum Marks : B.Sc. : 40, B.A. : 27

Note : Attempt *Five* questions in all, selecting *one question* from each Section. Q. No. 1 is *compulsory*.

(Compulsory Question)

1. (a) Find the magnitude and direction of velocity of a particle moving along the curve $x = at^2$, $y = 2at$. 2(1)

(b) The distances of a particle moving with S.H.M. from mean position are a, b, c at the end of 3 successive seconds. Find the periodic time. 1½(1)

(c) A body of mass 450 kg. is acted upon by a force 5 N for 15 minutes. If the particular starts from rest, find its velocity at the end of this period. 1½(1)

(d) Find the work done in stretching an elastic string of natural length l and modulus λ from tension T_1 to tension T_2. 1½(1)

(e) Find the orbit of a particle moving in a plane under a central force varing inversely as square of the distance from the fixed point. 1½(1)

Section I

2. (a) Derive the expressions for tangential and normal components of velocity of a particle moving in a plane curve. 4(3)

(b) A ship steams due west with a velocity 15 km/h relative to the current which is flowing at the rate of 6 km/h due south. What is the velocity of a train going north at a rate of 30 km/h relative to ship ? 4(2½)

3. (a) A particle executing simple harmonic motion travels a distance x in the first second after starting from rest. In the next second, it travels a distance y in the same direction. Find the amplitude of the motion. 4(3)

(b) One end of an elastic string whose modulus of a elasticity is λ and natural length l is tied to a fixed point on a smooth horizontal table and the other tied to a mass m lying on the table. The particle is pulled to a distance where extension of the string is a; and let go. Describe the character of motion. 4(2½)

Section II

4. (a) A bucket of water weighing 160 lbs. being raised from a well 100 ft deep with a uniform force of 200 lbs wt. With what acceleration will it move and in what time does it take to reach the top of the well ?

(b) A mass of 6 kg. is drawn across a rough horizontal table by means of a string passing over a smooth pulley fixed at the edge of the table and supports a mass of 2 kg. at the other end. If the coefficient of friction be 5/12, find the acceleration of the masses.

5. (a) State and prove the principle of conservation of energy.

(b) A spider hangs from the ceiling by a thread of modulus of elasticity equal to its weight. Show that it can climb to the ceiling with an expenditure of work equal to only ¼th of what would be required if the thread were inelastic ? 4(2½)

Section III

6. (a) A heavy particle is allowed to slide down a smooth vertical circle of radius $27a$ from rest at the highest point. Show that on leaving the circle it moves in a parabola of latus rectum $16a$. 4(3)

(b) A particle slides down a rough curve under gravity in a vertical plane. Discuss the motion. 4(2½)

7. (a) From the top of a cliff, 29.4 m high, a stone is thrown so that it starts with a velocity of 49 m per second and at an angle of 30° with the horizontal. Find where it strikes the ground at the bottom of cliff. 4(3)

(b) A body is projected at an angle α to the horizon so as to clear two walls of equal height 'a' at a distance $2a$, from each other. Show that the range is equal to $2a\cot\frac{1}{2}\alpha$. 4(2½)

Section IV

8. (a) A particle moves in an ellipse under a force which is directed towards its focus. Find its law of force and velocity at any point of its path. 4(3)

(b) If the central force varies as the cube of the distance from a fixed point, then find its orbit. 4(2½)

9. (a) The greatest and least velocities of a certain planet in its orbit round the sun are 30 km/sec. and 28.8 km/sec. respectively. Find the eccentricity of the orbit. 4(3)

(b) Find the acceleration of a particle in terms of cylindrical polar co-ordinates. 4(2½)