

Roll No. ....

(12/15-1)

**5002**

**B. Com. (Gen./Voc.) EXAMINATION**

(For Batch 2011 & 2012 Only)

(First Semester)

**BUSINESS MATHEMATICS-I**

**BC-102**

*Time : Three Hours*

*Maximum Marks : 80*

**Note :** Attempt *Five* questions in all. Q. No. 1 is compulsory. All questions carry equal marks.

1. (a) If  $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ , show that :

$$2x \frac{dy}{dx} + y = 2\sqrt{x}$$

(b) If  $z = e^{ax-by} f(ax+by)$ , prove that :

$$b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz$$

(c) Divide 14 into two parts such that their product is maximum.

(d) Evaluate :

$$\int \frac{2x-1}{2x^2-2x+1} dx$$

(e) Evaluate the area between the curve  $y = x^2$ , x-axis and the lines  $x = 0$  and  $x = 2$ .

(f) Define scalar matrix and diagonal matrix.

(g) Prove without expanding that :

$$\begin{vmatrix} 9 & 9 & 12 \\ 1 & -3 & -4 \\ 1 & 9 & 12 \end{vmatrix} = 0$$

(h) If  $A = \begin{bmatrix} 3 & -4 \\ 7 & 8 \end{bmatrix}$ , show that  $A - A'$  is a skew-symmetric matrix, where  $A'$  is the transpose of matrix  $A$ .

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2. (a) Differentiate the following functions w.r.t. 'x' :

(i)  $x^x + x^{1/x}$

(ii)  $x^x + x^{\log x}$

(b) If  $x^p y^q = (x + y)^{p+q}$ , then prove that :

$$\frac{d^2 y}{dx^2} = 0$$

3. (a) If  $u = \frac{xy}{x+y}$ , then show that :

$$x \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$

(b) Verify Euler's theorem for the function :

$$\frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$$

4. (a) An open tank with square base of side  $x$  metres and vertical height  $h$  meters is to be constructed so as to contain  $C$  cubic

metres of water. Show that the expenses on lining the inside of the tank with lead would be least if  $h = \frac{x}{2}$ .

(b) Minimize the cost for a firm with cost function  $C = 3x^2 + 5xy + 6y^2$  subject to the production constraint  $5x + 7y = 732$ .

5. (a) Evaluate :

$$\int \frac{x}{\sqrt{(x+1)(x-2)}} dx$$

(b) Evaluate :

$$\int (2x+3)\sqrt{x^2+4x+3} dx$$

6. (a) Demand and supply functions are  $p_d = (12 - 2x)^2$  and  $p_s = 56 + 4x$  respectively. Determine consumer's surplus under monopoly (so as to maximize the profit) and the supply function is identified with the marginal cost function.

(b) If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ , show that :

$$A^3 - 23A - 40I = 0$$

7. (a) Show that :

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

(b) Using determinants, solve the following system of linear equations :

$$\begin{aligned} x + y + z + 1 &= 0 \\ x + 2y + 3z + 4 &= 0 \\ x + 3y + 4z + 6 &= 0 \end{aligned}$$

8. (a) If  $A = \begin{bmatrix} 1 & 2 & 4 \\ 6 & 8 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ , express A as the sum

of a symmetric and a skew-symmetric matrix.

(b) Find the adjoint of the matrix

$A = \begin{bmatrix} 1 & 4 & 5 \\ 3 & 2 & 6 \\ 0 & 1 & 0 \end{bmatrix}$  and verify the theorem :

$$A(\text{adj. } A) = (\text{adj. } A) A = |A|I_3$$

9. (a) Find the inverse of the matrix

$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$  and verify that :

$$A.A^{-1} = A^{-1}.A = I$$

(b) Solve the equations by matrix method :

$$\begin{aligned} x + y + z &= 6 \\ x - y + z &= 2 \\ 2x + y - z &= 1 \end{aligned}$$