Roll No. .....

(06/22-II)

53211

B. Sc. EXAMINATION

(4 Year Programme)

(For Batch 2021 & Onwards)

(Second Semester)

MATHEMATICS

B.Sc./Maths/2/CC3

Number Theory and Trigonometry

Time : Three Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) If $(a, b) = 1$ and c/a, then prove that $(c, b) = 1$.

(b) Find remainder when 2^{20} is divided by 7.

(c) Find $d(n)$ and $\sigma(n)$ for $n = 270$.

(d) Find all values of $(1 + i)^{2/3}$.

(e) Find the general value of log (-5).

2×5=10

Unit I

2. (a) Prove that an integer is divisible by 3 iff the sum of its digits is divisible by 3. **8**

(b) Show that there are infinitely many primes of the form $6n + 5$. **7**

3. (a) If $(a, m) = 1$ i.e. a and m are co-prime. then linear congruence $ax \equiv b \pmod m$ has a unique solution. **8**

(b) If P is prime, show that :

$$2(p-3)! + 1$$

is a multiple of p.

Unit II

4. (a) Show that $\phi(15^K) = \phi(15) \times 15^K$, where K is a positive integer. 8

(b) Show that 2, 4, 6. ..... $2m$ is a CRS (mod m) if m is odd. 7

5. (a) Find Highest power of 9 which divides 365!. 8

(b) Show that 3 is a quadratic non-residue of 31. 7

Unit III

6. (a) If :

$$x_r = \cos\left(\frac{\pi}{2^r}\right) + i \sin\left(\frac{\pi}{2^r}\right).$$

prove that :

$$x_1 x_2 x_3 \ldots\ldots \infty = -1.$$ 8

(b) Find all the values of $(1+\sqrt{3}i)^{3/4}$ and show that their product is 8. 7

7. (a) If $z = x + iy$, find real and imaginary parts of $\frac{\cos z}{z+1}$. 8

(b) If :

$$x + iy = \cos(u + iv),$$

show that :

$$(1+x)^2 + y^2 = (\cosh v + \cos u)^2.$$ 7

Unit IV

8. (a) Show that :

$$i^{\alpha+i\beta} = e^x (\cos y + i \sin y)$$

where :

$$x = -\frac{1}{2}(4n+1)\pi\beta,$$

$$y = \frac{1}{2}(4n+1)\pi\alpha.$$ 8

7

(b) Prove that :

$$\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{27}{11}\right).$$

9. (a) Prove that : 8

$$\sin^{-1}(\operatorname{cosec}\theta) = [2n + (-1)^n]\frac{\pi}{2} + i(-1)^n \log \cot\frac{\theta}{2}.$$

(b) Prove that : 7

$$\frac{\pi}{4} = \left(\frac{2}{3} + \frac{1}{7}\right) - \frac{1}{3}\left(\frac{2}{3^3} + \frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5} + \frac{1}{7^5}\right) + \ldots\ldots\infty$$