Roll No.....

(05/19-I)

11711

M. Sc. (5 Years) EXAMINATION

(For Batch 2017 & Onwards)

(Tenth Semester)

MATHEMATICS

MTHCC-5001

Functional Analysis

Time : Three Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** from Section I is compulsory.

Section I

1. (a) Define Adjoint Operator with the help of an example.

(b) State Parseval's Identity.

(c) Write down the conjugate space of k_n as $l_p^{(n)}$.

(d) State Uniform Bondedness Theorem.

(e) State Riesz Representation Theorem.

(f) Define Hilbert Space.

(g) State Hahn Banach Theorem.

Section II

2. Show that the linear space C [C, 1] of all real valued continuous functions on [0, 1] is complete normed space under the norm : **14**

$$\| f \| = \int_0^1 | f(x) | dx \quad \forall f \in C[0,1].$$

3. (a) State and prove F. Riesz Lemma. **8**

(b) Show that scalar multiplication is jointly continuous in X where X is a normed linear space over the scalar field F. **6**

Section III

4. State and prove Riez Representation Theorem for bounded linear functionals on C $[a, b]$. **14**

5. (a) Show that the space C [0, 1] is not reflexive. **8**

   (b) Give an application of Hahn Banach Theorem. **6**

Section IV

6. (a) Prove that in a finite dimensional space, the notion of weak and string convergence are equivalence. **12**

   (b) State Closed Graph Theorem. **2**

7. (a) Show that if P is a projection on a Banach Space B as if M and N are its range and null space, then M and N are closed linear subspaces of B such that $B = M \oplus N$. **7**

   (b) State and prove Schwartz's Inequality. **7**

Section V

8. (a) Show that any *two* complete orthonormal sets in a Hilbert space H have the same cardinal number. **7**

   (b) State and prove Parseval's Identity. **7**

9. (a) State and prove under what conditions sum or product of two normal operators are normal. **7**

   (b) Show that the unitary operators on H form a group. **7**