

Roll No. ....

(07/21-II)

**11659**

**M. Sc. (2 Year) EXAMINATION**

(For Batch 2019 & Onwards)

(Second Semester)

MATHEMATICS

MTHCC-2202

Measure & Integration Theory

*Time : Three Hours*

*Maximum Marks : 70*

**Note :** Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

1. (a) If  $M^*(A) = 0$ , then prove that  $M^*(A \cup B) = M^*(B)$ .
- (b) Define measurable set and prove that difference of two measurable set is measurable.

- (c) Prove that characteristic function of a set  $A$  is measurable if  $A$  is measurable.
- (d) Show that every function defined on a set of measure zero is measurable.
- (e) Give an example to show that the Lebesgue integral of a non zero function can be zero.
- (f) Prove that in Lebesgue dominated convergence theorem, the existence of dominant function is sufficient condition, not necessary. <https://www.cdluonline.com>
- (g) Prove that every function of bounded variation is bounded.

**Unit I**

2. (a) Prove that outer measure of countable set is zero. Is the converse true if not provide suitable example ?
- (b) Show that a set is Lebesgue measurable if and only if its complement is measurable.

3. (a) If  $E_1, E_2, E_3, \dots$  are pairwise disjoint measurable sets and  $E = E_1 \cup E_2 \cup E_3 \cup \dots$ , then show that  $E$  is measurable

$$\text{and } m(E) = \sum_{r=1}^{\infty} m(E_r).$$

- (b) Prove that the set of type  $F_\sigma$  and  $G_\delta$  are measurable sets.

## Unit II

4. (a) Show that a function is simple if and only if it is measurable and assumes a finite number of values.

~~(b)~~ State and prove Lusin's theorem.

5. (a) Let  $f$  be a function defined on measurable set  $E$ . Then  $f$  is measurable iff for any open set  $O$  in  $\mathbb{R}$ ,  $f^{-1}(O)$  is measurable set.

~~(b)~~ State and prove F. Riesz's theorem.

## Unit III

6. (a) Show that the function  $f$  defined on the interval  $[a, b]$  by :

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$$

is Lebesgue integrable, but not Riemann integrable.

~~(b)~~ State and prove Fatou's Lemma.

7. (a) Verify the result of bounded convergence theorem for the function :

$$f_n(x) = \frac{nx}{1+n^2x^2}, 0 \leq x \leq 1$$

- (b) State and prove necessary and sufficient condition of Lebesgue integrability.

## Unit IV

8. (a) State and prove Jordan de-composition theorem.

(b) Show that the function  $f$  defined on  $[0, 1]$  by :

$$f(x) = \begin{cases} x \cdot \sin \frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

is not of bounded variation.

9. State and prove Lebesgue differentiation theorem.

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