

Roll No.

(07/22-II)

15215

M. Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Second Semester)

MATHEMATICS

MSC/MATH/2/DSC1

Methods of Applied Mathematics

Time : Three Hours

Maximum Marks : 70

Note : Attempt *Five* questions in all, selecting the compulsory question ($5 \times 2 = 10$) and *one* question from each Unit of 15 marks each.

Compulsory Question

1. (a) Define arc length and volume element.
- (b) State Parseval's identity for Fourier transform.

- (c) Define co-variant of a vector.
- (d) Define Hankel inverse transform.
- (e) Show that the extremal of the functional

$$\int_a^b \left(y + \frac{y^3}{3} \right) dx \text{ does not exist.}$$

Unit I

2. (a) Express $\nabla \vec{f}$ in orthogonal curvilinear coordinates and deduce it in spherical coordinates.
(b) Prove that spherical coordinate system is orthogonal.
3. (a) Express the velocity \vec{v} and \vec{a} acceleration of a particle in cylindrical coordinates.
(b) Represent $\vec{F} = y\hat{i} - 2x\hat{j} + z\hat{k}$ in cylindrical co-ordinates.

Unit II

4. (a) Find Fourier transform of function :

$$f(t) = \begin{cases} t, & |t| \leq a \\ 0, & \text{otherwise} \end{cases}$$

- (b) Use Parseval's identity to prove :

$$\int_0^\infty \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a+b)}$$

5. (a) State and prove Parseval's identity for Fourier transform.

- (b) Solve, using Fourier transform :

$$\frac{\partial u}{\partial t} = \frac{2\partial^2 u}{\partial x^2}$$

$u(0, t) = 0$, $u(x, 0) = e^{-x}$ and $u(x, t)$ is bounded, where $x > 0, t < 0$.

Unit III

6. (a) Find Hankel transform of $H_0\left[\frac{\sin ax}{x}\right]$.

- (b) Find :

$$H_n\left[x^{n-1} \frac{d}{dx}(x^{1-n} f(x))\right]$$

7. (a) Prove that :

$$H_n\left[\frac{J_{n+1}(ax)}{x}\right] = \frac{\xi^n}{a^{n+1}} H(a - \xi)$$

where $H(a, \xi)$ is Heaviside unit step function.

- (b) Find the Hankel transformation of

$$\frac{d^2}{dx^2}(f(x)).$$

Unit IV

8. (a) Show that the extremal for :

$$I = \iint_D \left[\left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \right] dx dy$$

is a solution of Laplace equation
 $\nabla^2 z = 0$.

(b) Find extremal of the functional :

$$J(y, z) = \int_0^{\pi/2} (y'^2 + z'^2 + 2yz) dx$$

where $y(0) = 0, y\left(\frac{\pi}{2}\right) = 1, z(0) = 0,$

$$z\left(\frac{\pi}{2}\right) = -1.$$

9. (a) Find the geodesics on the surface of a sphere.
(b) Find the extremals of the functional :

$$I(y) = \int_1^2 \frac{\sqrt{1+y'^2}}{x} dx, y(1) = 0, y(2) = 1$$