Roll No. ....

(07/21-II)

11782

M. Sc. (2 Year) EXAMINATION

(For Batch 2019 & Onwards)

(Second Semester)

MATHEMATICS

MTHCC-2204

System of Differential Equations

Time : Three Hours *Maximum Marks :* 70

Note : Question No. **1** is compulsory. Attempt *Five* questions in all, selecting *one* question from each Unit including compulsory question.

(Compulsory Question)

1. (a) If $A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 4 & -2 \end{bmatrix}$, find the determinant

of the fundamental matrix ϕ satisfying $\phi(0) = E$.

(b) Prove that two different homogeneous systems cannot have the same fundamental matrix.

(c) Explain the concept of path approaching a critical point.

(d) Find the solution of the linear autonomous system $\frac{dx}{dt} = x$, $\frac{dy}{dt} = x + y$ satisfying the condition $x(u) = e$, $y(u) = ue$.

(e) Explain Floquet Theory.

(f) Define limit set of an orbit.

(g) State Poincare Bendixson Theorem.

7×2=14

Unit I

2. (a) Find the necessary and sufficient condition for n solutions of the linear system $x' = A(t)x$ to be linearly independent. 7

(b) If B is a non-singular matrix, then show that there exists a matrix A such that $e^{A} = B$. 7

3. (a) Derive Abel-Liouville formula for a linear homogeneous system. 7

(b) State and prove the relationship between fundamental matrices of a linear homogeneous system and its adjoint system. 7

Unit II

4. (a) State and prove Abel-Liouville formula for an nth order homogeneous linear differential equation. 7

(b) Find the solution and the fundamental matrix of the linear system with constant coefficient $x' = Ax$, where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. 7

5. (a) State and prove Representation Theorem for a linear system with periodic coefficients. 7

(b) Derive variance of constant formula for non-homogeneous linear system. 7

Unit III

6. (a) Define a plane autonomous system. What are different types of critical points ? 7

(b) If two roots of the characteristic equation for a linear autonomous system are conjugate complex with real part not zero, then find the nature of critical point and check its stability. 7

7. (a) Determine the nature of critical point of the linear system :

$$\frac{dx}{dt} = 2x - 4y, \quad \frac{dy}{dt} = 2x - 2y$$

and check its stability. 7

(b) Find all the real critical points of the non-linear system :

$$\frac{dx}{dt} = 8x - y^2, \quad \frac{dy}{dt} = -6y + 6x^2$$

and determine the type and stability of each of the critical point. 7

Unit IV

8. (a) Define Lyapunov function for a non-linear autonomous system. Construct a Lyapunov function for the system :

$$\frac{dx}{dt} = -x + y^2, \quad \frac{dy}{dt} = -y + x^2$$

and use it to determine the stability of the critical point (0, 0) of this system. 7

(b) Examine the critical points of the non-linear differential equation :

$$\frac{d^2x}{dt^2} = x^2 - 4x + \lambda,$$ λ being a parameter.

Also find the critical values of the parameter. 7

9. (a) Explain Bendixson criterion for non-existence of limit cycles of a non-linear autonomous system. Also, provide a suitable example. 7

(b) Define the following : 7

(i) Index of a curve

(ii) Half-path for a non-linear system.