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11668**M. Sc. (2 Year) EXAMINATION**

(For Batch 2018 & Onwards)

(Third Semester)

MATHEMATICS

MTHCE-2303

Integral Equations

: *Three Hours* *Maximum Marks : 70*: Attempt *Five* questions in all, selecting *one* question from each and Unit. Q. No. 1 is compulsory. All questions carry equal marks.**(Compulsory question)** $2 \times 7 = 14$

- (a) Show that the function $y(x) = xe^x$ is a solution of the Volterra integral equation $y(x)$.

- (b) Determine the eigen values and eig function of the homogeneous integ equation :

$$y(x) = \lambda \int_0^{2\pi} \sin x \sin t y(t) dt$$

- (c) Define integral equations with example
 (d) Define Cauchy general and Cauchy principal value.
 (e) Define Riemann Hilbert problem.
 (f) What are basic four properties of Green function ?
 (g) Find second approximation of the method of successive approximation for 1 integral equation :

$$y(x) = x - \int_0^x (x-t) y(t) dt$$

when $y_0(x) = x$.

Unit I

- (a) Solve the following Volterra integral equation of the first kind : 7

$$f(x) = \int_0^x e^{x-t} y(t) dt, f(0) = 0$$

- (b) Solve the equation :

$$y(t) = e^{-t} - 2 \int_0^t \cos(t-x) y(x) dx$$

by using Laplace transform. 7

- Discuss solution of Volterra integral equation of the second kind by successive substitutions. 14

Unit II

1. Solve the following integral equation and discuss all its possible cases : 14

$$y(x) = f(x) + \lambda \int_0^1 (1-3xt) y(t) dt$$

5. (a) Solve the integral equation :

$$y(x) = 1 + \lambda \int_0^1 (x+t) y(t) dt$$

by the method of successive approximation upto second order for $y_0(x) = 1$. 7

- (b) Solve :

$$y(x) = f(x) + \lambda \int_{-1}^1 (xt + x^2 t^2) y(t) dt.$$

Find its resolvent kernel also. 7

Unit III

6. Find solution of the Cauchy type singular integral equation : <https://www.cdluonline.com>

$$ay(t) = f(t) - \frac{b}{\pi i} \int_C^* \frac{y(\xi)}{\xi - t} d\xi,$$

where a, b are complex constants, $y(\xi)$ is holder continuous function and C is a regular closed contour. 14

7. Find the solution of Hilbert type singular integral equation of the second kind :

$$ay(x) = f(x) - \frac{b}{2\pi} \int_0^{*2\pi} y(t) \cot\left(\frac{t-x}{2}\right) dt,$$

where a and b are complex constants. 14

Unit IV

8. (a) Find the Green function for : 7

$$y'' = 0; y(0) = y'(1), y'(0) = y(1)$$

- (b) Construct Green function for the differential equation $xy'' + y' = 0$ for the following conditions, $y(x)$ is bounded as $x \rightarrow 0$ and $y(1) = \alpha y'(1)$, $\alpha \neq 0$. 7

9. Transform the problem $\frac{d^2 y}{dx^2} + y = x$,

$y(0) = 1, y'(1) = 0$ to a Fredholm integral equation using Green's function. 14