11671

M. Sc. (2 Year) EXAMINATION

(For Batch 2017 & Onwards)

(Fourth Semester)

MATHEMATICS

MTHCC-2401

Functional Analysis

me : Three Hours *Maximum Marks : 70*

ote : Attempt *Five* questions in all, selecting *one* question from each Unit including compulsory question. All questions carry equal marks.

Compulsory Question

. (a) Show that each of the linear spaces R and C with the norm defined by $\|x\| = |x|$ is a normed space. 2

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(b) Define bounded linear functional .

(c) Define B(X, Y). .

(d) Define an algebra in a linear space .

(e) Define natural imbedding of N into N** .

(f) Show that inner product in a pre-Hilbert space is jointly continuous. .

(g) Define positive operator. .

Unit I

2. (a) A normed linear space is complete iff every absolutely convergent series in X is convergent. .

(b) A subspace Y of a Banach space X is complete if and only if the set Y is closed in X. https://www.cdluonline.com 7

3. (a) If M is a closed linear subspace of a normed linear space N and if $T : N \to N/M$ defiend by $T(x) = x + M$. Show that T is continuous linear transformation for which $\|T\| \leq 1$. 7

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(b) Let E and F be two normed linear spaces. Then they are topologically isomorphic iff $\exists$ m, M and a linear mapping $T : E \to F$ which is 1 1 and onto s.t. $m\|x\| \leq \|Tx\| \leq M\|x\| \; \forall \; x \in E$. 7

Unit II

(a) Show that l_p space is reflexive. 7

(b) Show that C[0, 1] is not regular. 7

State and prove Hahn-Banach theorem 14

Unit III

State and prove open mapping theorem. 14

(a) A Banach space is a Hilbert space iff parallelogram law holds. 7

(b) Let M be a closed linear subspace of a Hilbert space H, let $x \notin M$, and let d be the distance from x to M. Then there exists a unique vector y_0 in M s.t. $\|x - y_0\| = d$. 7

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Unit IV

8. Show that every Hilbert space is reflexive.

9. State and prove Bessel's inequality.

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