Roll No. ........................

(03/20-II)

# 11671

## M. Sc. (2 Year) EXAMINATION

(For Batch 2017 & Onwards)

(Fourth Semester)

MATHEMATICS

MTHCC-2401

Functional Analysis

*Time : Three Hours* *Maximum Marks : 70*

**Note :** Q. No. 1 is compulsory. Attempt *Five* questions in all, selecting *one* question from each Unit including compulsory question.

1. (a) If N is a normed linear space, then $\left|\|x\|-\|y\|\right| \leq \|x-y\|$ for all $x, y \in N$. 2

(b) Define conjugate operator. 2

(c) Define isometric isomorphism. 2





(d) Prove that $l'$ is not reflexive.

(e) Define a closed linear transformation.

(f) In a pre-Hilbert space, every Cauchy sequence is bounded.

(g) In N is a normal operator on H then :

$$\|N^2\| = \|N^2\|$$

### Section I

2. (a) Let N be a normed linear space. The closed unit Ball $B = \{x \in X : \|x\| \leq 1\}$ in N is compact if and only if N is finite dimensional.

(b) State and prove F. Riesz's lemma.

3. If N and N' are normed linear spaces, then the set $B(N, N')$ of all continuous L.T. of N into N' is itself a normed linear space with respect to the pointwise linear operations and the norm defined by :

$\|T\| = \sup\{\|Tx\| : \|x\| \le 1\}$

Further if N' is a Banach space, then B(N, N') is also a Banach space. 14

## Section II

4. (a) Let M be a closed linear subspace of a normed linear space N and let $x_0$ be a vector not in M, then there exists a functional F in N* such that $F(M) = \{0\}$ and $F(x_0) \neq 0$. 7

(b) Show that $C[0,1]$ is not reflexive. 7

5. Let N be an arbitrary normed linear space. Then for each vector $x \in N$, the scalar valued function $F_x$ defined by :

$$F_x(f) = f(x) \; \forall f \in N^*$$

is a continuous linear functional in N** and the mapping $x \to F_x$ is then an isometric isomorphism of N inti N**. 14





## Section III

6. (a) A linear transformation is closed iff its graph is a closed subspace. 7

(b) In a finite dimensional space, the notions of weak and strong convergence are equivalent. 7

7. (a) A Banach space is a Hilbert space iff parallelogram law holds. 7

(b) If M is a proper closed linear subspace of a Hilbert space H, then there exists a non-zero. 7

## Section IV

8. (a) Let H be a Hilbert space and let $\langle e_i \rangle$ be an orthonormal set in H. then the following conditions are all equivalent to one another :

(i) $\langle e_i \rangle$ is complete

(ii) $x \perp \langle e_i \rangle \Rightarrow x = 0$

(iii) If $x$ is any arbitrary vector in H, then $x = \sum (x, e_i) e_i$.

(iv) If $x$ is any arbitrary vector in H, then $\|x\|^2 = \sum |(x, e_i)|^2$. 7

(b) The self adjoint operators in B(H) form a closed real linear subspace of B(H) and therefore a real Banach space-which contains the identity transformation. 7

9. (a) If P is a projection on H with range M and null space N, then $M \perp N, \Leftrightarrow P$ is self-adjoint and in this case $N = M^{\perp}$. 7

(b) If $N_1$ and $N_2$ are normal operators on a Hilbert space H with the property that either commutes with the adjoint of the other, then $N_1 + N_2$ and $N_1 N_2$ are normal. 7