Roll No. .....

(01/22-II)

11707

M. Sc. (5 Years) EXAMINATION

(For Batch 2018 & Onwards)

(Ninth Semester)

MATHEMATICS

MTHCC-5902

Fluid Mechanics

Time : Three Hours *Maximum Marks : 70*

Note : Attempt *Five* questions in all including Q. No. 1 which is compulsory. Select *one* question from each Unit. All questions carry equal marks.

1. (a) Define uniqueness theorem of kinetic energy of liquid.

   (b) Defne boundary surface and write condition for it.

   (c) What do you mean by isotropic fluid and Beltrami vector.

   (d) Find equation of streamline for the flow field defined by :

$$\vec{q}\left(x\left(1-t^2\right), y(1+t), 0\right)$$

   (e) Define source, sink and doublet.

   (f) Show that Stokes stream function is constant on streamlines.

   (g) Describe axisymmetric flows.

Unit I

2. (a) Define circulation and vorticity. Prove that the divergence free of vorticity leads to constant circulation in the flow field.

   (b) State and prove equation of continuity and find condition of incompressibility.

3. (a) Find circulation and the vorticity vector of the flow field :

$$\vec{q} = x^2 y\hat{i} - y^2\hat{j}$$

(b) A velocity field in a particular flow is given by :

$$\vec{q} = x^2 y\hat{i} - xy^2\hat{j} + z\hat{k}$$

Calculate the acceleration, the angular velocity at the point (–1, –1, 1).

Unit II

4. (a) Derive Bernoulli's equation for the steady flow of an ideal fluid subject to conservative body forces.

(b) State and prove Kelvin's minimum energy theorem.

5. (a) State Newton's second law and derive Euler's equation of motion of an ideal incompressible fluid flow.

(b) Derive equation of vorticity and explain its advantage.

Unit III

6. (a) A sphere of density σ moves with velocity U in an infinite mass of liquid of density ρ at rest at infinity, then find the ratio in which effect of external forces are reduced due to presence of the liquid.

(b) Find image of a source with respect to a sphere.

7. (a) Find potential function and stream function for the liquid streaming past a fixed sphere and obtain the lines of flow relative to the sphere.

(b) The liquid is filled in the space between concentric spheres then find kinetic energy of the liquid when the spheres start moving with applied impulsive forces.

Unit IV

8. (a) Define stream function potential function for a cylindrical coordinate system and show that lines of constant φ intersect lines of constant ψ at right angles.

(b) State and prove Milne-Thomson circle theorem and use it to find the complex potential function for the liquid streaming past a circular cylinder with circulation.

9. (a) Show that for two-dimensional, irrotational, incompressible flow the stream function satisfies Laplace's equation.

(b) State and prove Blasius theorem.