

Roll No.

(01-22-II)

11708

M. Sc. (5 Years) EXAMINATION

(For Batch 2018 & Onwards)

(Ninth Semester)

MATHEMATICS

MTHCC-5903

Integral Equations

Time : Three Hours *Maximum Marks : 70.*

Note : Attempt Five questions in all, selecting one question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Define integral equations.
- (b) Define Neumann series for volterra equations.
- (c) Define Boundary value problem with example.

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- (d) Define Cauchy principal value for integrals.
- (e) Define singular integral equation with example.
- (f) Write Fredholm resolvent Kernel as a ratio of two series.
- (g) Write basic properties of Green's function.

Unit I

2. Discuss the method of successive substitution to solve Volterra integral equation of second kind in detail. 14
3. (a) Find the resolvent Kernel of the Volterra integral equation with the Kernel $K(x,t) = e^{x-t}$. 7
- (b) Using the method of successive approximations, solve the integral equation $y(x) = 1 + \int_0^x (x-t)y(t)dt$, taking $y_0(x) = 1$. 7

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Unit II

4. (a) Solve the homogeneous Fredholm integral equation of the second kind :

$$y(x) = \lambda \int_0^{2\pi} \sin(x+t) y(t) dt.$$

(b) Solve :

$$y(x) = (1+x)^2 + \int_{-1}^1 (xt + x^2 t^2) y(t) dt. \quad 7$$

5. (a) Show that the integral equation :

$$y(x) = f(x) + \frac{1}{\pi} \int_0^{2\pi} \sin(x+t) y(t) dt$$

possesses no solution for $f(x) = x$, but it possesses many solutions when $f(x) = 1$.

(b) By iterative method, solve :

$$y(x) = 1 + \lambda \int_0^\pi \sin(x+t) y(t) dt. \quad 7$$

Unit III

6. Solve the Abel integral equation :

$$f(x) = \int_a^x \frac{y(t) dt}{a[h(x) - h(t)]^\alpha}, \quad 0 < \alpha < 1$$

where $h(t)$ is a strictly monotonically increasing and differentiable in (a, b) , $h'(t) \neq 0$ and hence solve :

$$f(x) = \int_a^x \frac{y(t) dt}{a(\cos t - \cos x)^\frac{1}{2}}, \quad 0 \leq a < x < b \leq \pi. \quad 14$$

7. State and prove Poincare Bertrand transformation formula. 14

Unit IV

8. Discuss the method of variation of parameters to construct the Green function for a non-homogeneous linear second order boundary value problem. 14

9. (a) Construct the Green's function for the homogeneous boundary value problem

$$\frac{d^4y}{dx^4} = 0, \quad y(0) = y'(0) = y(1) = y'(1) = 0$$

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- (b) Using Green's function, solve the boundary value problem :

$$y'' + y = x, \quad y(0) = y\left(\frac{\pi}{2}\right) = 0. \quad 7$$